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Effect of Dissipation on Excitation of Beam Wave with Negative Energy

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The effect of dissipation on the instability of overlimiting electron beam caused by excitation of the beam wave with negative energy and transition of this instability to that of dissipative type as the level of dissipation increases is investigated. Growth rates are obtained. The interplay of two effects both leading to excitation of the beam wave with negative energy leads to more critical (as compared with the case of dissipative instability of sublimiting electron beam) dependence of the growth rate on dissipation. An equation describing the influence of dissipation on space-time development of the fields is derived and solved. Analysis of the solution is presented

Two basic trends of high power microwave electronics are following: increasing power and frequency of output radiation [1,2]. First trend leads to increasing of beam current. In plasma-filled devices it can several times exceed limiting current in vacuum devices. But the physical character of beam-plasma interaction changes. Instability of overlimiting electron beam is due not to induced radiation of system proper waves, but either to excitation of the beam wave with negative energy, or to aperiodic modulation of the beam space charge in medium with negative dielectric constant [3-5]. The first type of overlimiting electron beam instability realizes in beam-plasma systems that are no uniform in cross section. Moreover many new physical phenomena exhibit themselves if system consists of spatially separated beam and plasma [6]. Change of the physical character of beam plasma interaction results in change of energy transfer from beam electrons to excited oscillations.

At changing over to higher frequencies, for effective interaction of beam electrons with slowing down space harmonics the beam should be driven into resonators' walls [2]. The skin depth decreases, and the Q-quality of resonators falls down. Energy loss in the walls of resonators increases. Influence of dissipation on interaction of overlimiting electron beam with plasma may leads to an array of new phenomena that are significant for microwave generation because of dissipation also leads to excitation of the beam wave with negative energy. Dissipation is nothing else as a channel of energy withdrawal and such an excitation actually is dissipative beam instability. In systems with overlimiting electron beams the influence of dissipation is significant because (in contrary to sublimiting beams) actually the same mechanism of instability takes place without dissipation and dissipation intensify it. Influence of dissipation on the operation of microwave devices with overlimiting beam has not been considered yet. Superposition of the two effects that led to excitation of the beam wave with negative energy, results in critical effect of dissipation on the instability. The dependence of the growth rate on the parameter characterizing dissipation becomes more critical. The influence of dissipation on spatial-temporal development and mode structure of the instability is investigated. Gradual transition of this type of

overlimiting beam instability to that of dissipative type is elaborated.

Consider fully magnetized plasma filled waveguide penetrated by overlimiting relativistic electron beam. In such a system physical character of beam-plasma interaction essentially depends on transversal geometry. In the case of inhomogeneous beam and/or plasma the instability is due to excitation of beam wave with negative energy.

Generally say, rigorously treatment of the problem may not be developed based on the perturbation theory with small parameter. But in the case of spatially separated beam and plasma when the integral describing fields overlap is small, the effect may be considered analytically. In this case $p_b(\mathbf{r}_\perp)p_p(\mathbf{r}_\perp) = 0$ (functions $p_{p,b}(\mathbf{r}_\perp)$ represent the profiles of the plasma and beam densities in the waveguide cross-section; for homogeneous beam and plasma $p_{p,b}(\mathbf{r}_\perp)=1$ for thin annular beam $p_b = \delta(r-r_b)$, where δ is the Dirac function and r_b is the beam radius). In zero approximation the system may be described by solving of following two independent problems

$$\Delta_\perp E_{z\alpha} - \left(k^2 - \frac{\omega^2}{c^2} \right) [1 - p_\alpha(r_\perp)] \delta \varepsilon_\alpha E_{z\alpha} = 0 \quad (1)$$

with the conditions $E_{z\alpha} = 0$ on the metallic surfaces. Here $\alpha = p, b$ for plasma and beam respectively, $\delta \varepsilon_p = \omega_p^2 / \omega(\omega + i\nu)$; $\delta \varepsilon_b = \omega_b^2 \gamma^{-3} (\omega - ku)^{-2}$, Δ_\perp is the Laplace operator with respect to transversal coordinates, ω and k are the frequency and wave vector of perturbations E_z is the longitudinal electric field which is represented in a form $E_z(\mathbf{r}_\perp, z, t) = E_z(\mathbf{r}_\perp, \omega, k) \exp(-i\omega t + ikz)$, z is the coordinate along the waveguide axis, t is the time, $\omega_{p,b}$ are the Lengmuire frequencies for plasma and beam respectively, γ is the relativistic factor of beam electrons $\gamma = (1 - u^2/c^2)^{-1/2}$, u is the velocity of beam electrons, c is the speed of light. The zero order dispersion relations are

$$D_\alpha(\omega, k) = k_{z\alpha}^2 + \left(k^2 - \frac{\omega^2}{c^2} \right) (1 - \delta \varepsilon_\alpha) = 0 \quad (2)$$

where $k_{z\alpha}$ are determined by the proper function of zero

order problems respectively. If one applies the perturbation theory to this state he can obtain after cumbersome calculations the dispersion relation, which takes into account the beam-plasma interaction in first order approximation

$$D_p(\omega, k)D_b(\omega, k) = G\left[\left(k^2 - \frac{\omega^2}{c^2}\right)^2 \delta\epsilon_p \delta\epsilon_b\right] \quad (3)$$

Here $G > 0$ and it is nothing else as so-called geometrical factor of the space charge. It shows the overlap of the plasma and the beam fields and represents all specific properties of considered system. The factor D_b in the dispersion relation (3) coincides with the dispersion relation describing beam oscillations in magnetized waveguide in the case of full filling. In the simplest one dimensional limit the spectra of beam oscillations are given by well known simplest expression

$$\omega = ku \pm \Omega_b \quad (4)$$

where $\Omega_b = \omega_b \gamma^{-3/2}$. In general case $\Omega_b = \Omega_b(\omega, k)$ and the spectra take complicate form. Obvious expression for the spectra can be obtained if one neglects the biased current as compared with the high frequency beam current i.e. $k \ll k_{\perp} \gamma$

$$\omega_{\pm} = ku \pm \alpha^{1/2} / \gamma \quad (5)$$

where $\alpha = \omega_b^2 / k_{\perp b}^2 u^2 \gamma^3$. Non-potential character of the beam waves is intrinsic for high beam current only, comparable or higher than the limiting vacuum current. If one search the solutions of (3) for the form $\omega = ku + \delta$ dispersion relation takes the form

$$\left(x + \Delta + i \frac{v_0}{ku} f\right) \left(x^2 - \frac{\alpha}{\gamma^2}\right) = -\frac{G}{2\gamma^4} \alpha \frac{\omega_p^2}{k^2 u^4 / c^2 - \omega_p^2} \quad (6)$$

where

$$\Delta = \frac{k^2 u^2 - \omega_0^2}{2\gamma^2(\omega_p^2 - k^2 u^4 / c^2)} \quad f = \frac{\omega_p^2(1 - 2\beta^2 \gamma^2 x)}{2\gamma^2(k^2 u^4 / c^2 + \omega_p^2)}$$

v_0 is the effective collision frequency in plasma. The growth rate attains its maximum if (in absence of dissipation) $\Delta = -\sqrt{\alpha} / \gamma$ and is equal

$$\delta_{st} = (2\gamma)^{-1} (Gf)^{1/2} (\omega_b^2 / k_{\perp b}^2 u^2 \gamma^5)^{1/4} \quad (7)$$

If dissipation in the system is high enough ($v_0 \gg \delta$) the instability become of dissipative type with following growth rate

$$\delta_v = \frac{\omega_0}{v} (G/4\gamma^3) \alpha^{1/2} \omega_p^2 / (k^2 u^2 - \omega_0^2) \quad (8)$$

It can be written as $\delta_v = \delta_{st}^2 / v$

where $v = \text{Im} D_p / (\partial D_p / \partial \omega)_{\omega=\omega_0, k=k_0}$

Consider now in detail how the increasing in level of dissipation changes the character of instability and transform it. The development of an initial perturbation in the system and influence of the dissipation on the shape of induced waveform will show as the details of this transition. In so doing we represent the field E_z in form with slowly varying amplitude

$$E_z(\mathbf{r}_\perp, z, t) = E_0(\mathbf{r}_\perp, z, t) \exp(-i\omega_0 t + ik_0 z) \quad (9)$$

where the dependence $E_0(z, t)$ is weak in following mean

$$|\partial E_0 / \partial t| \ll \omega_0 E_0, \quad |\partial E_0 / \partial z| \ll k_0 E_0, \quad (10)$$

ω_0 and k_0 is solution of zero order equations (2). The transversal structure of the fields may be obtained in the normal way by expanding on series of the eigenfunctions of given system. Longitudinal structure of induced waveform is described by equation

$$\left(\frac{\partial}{\partial t} + v_b \frac{\partial}{\partial z}\right) \left(\frac{\partial}{\partial t} + v_p \frac{\partial}{\partial z} + v\right) E_0(z, t) = \delta_{st}^2 E_0(z, t) \quad (11)$$

where v_p and v_b are the group velocities of plasma and beam waves respectively. Solution of equation (11) gives us the following expression for slowly varying amplitude

$$E_0(z, t) = -\frac{J_0}{2\sqrt{\pi}} \frac{e^{\chi(z,t)}}{(u - v_0)^{1/2} \delta_{st}^{1/2} (ut - z)^{1/2}} \quad (12)$$

$$\chi(z, t) = \frac{2\delta_{st}}{u - v_p} \left\{ (z - v_p t)(v_p t - z) \right\}^{1/2} - v(v_p t - z)^{1/2} \quad J_0 = \text{const.}$$

This expression explicitly shows convective character of instability in separated beam-plasma system. Unstable perturbation ranges as $v_p \leq v \leq v_b$. Growth rate in the peak is equal to (7). The peak of the wave train moves at average velocity $w_p = (v_p + v_b)/2$. Dissipation suppresses slow perturbations. The threshold velocities $v_{th} = (\lambda v_b + v_p) / (1 + \lambda)$ where $\lambda = v^2 / 4\delta^2$. Dynamics of instability in presence of dissipation may be obtained by analyzing the following equation

$$\lambda(v_b t - z)(z - v_p t) = (w_p t - z)^2 \quad (13)$$

Exact solution of the equation (11) leads to following expression for point of peak

$$z = \frac{t}{2} \left\{ v_b + v_p + \lambda'(v_b - v_p) \right\} \quad \lambda' = \sqrt{\lambda / (1 + \lambda)} \quad (14)$$

Substitution of (14) into χ (see (12)) leads to following maximal growth rate depending on dissipation level

$$\delta_{\max}(v) = \delta_{st} \left(\sqrt{1 + x^2} - x \right) \quad \text{where } x = v / \delta_{st}$$

In the limit $v \gg \delta_{st}$ the wave train significantly shortens and the growth rate in the peak is equal δ_v (9) i.e. the dissipative instability of overlimiting electron beam develops and its growth rate has more critical dependence on dissipation

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